LIBERTUS FROMONDUS' ESCAPE FROM THE LABYRINTH OF THE CONTINUUM (1631)

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Abstract

This article is concerned with a previously little studied work, the *Labyrinthus sive de compositione continui liber unus* (1631), in which Libertus Fromondus attacked the atomistic theory, which at the time was finding supporters at the University of Louvain. I try to identify Fromondus' sources and polemical targets, to summarise his mathematical and physical arguments against atomism, and to understand his nominalist solution to the problem of the composition of the continuum. Moreover, I situate the *Labyrinthus* in the context of seventeenth-century theories of matter and motion and attempt to provide new evidence of Fromondus' possible influence on Galileo and of his undeniable influence on Leibniz.

Keywords: Libertus Fromondus – Galileo Galilei – Gottfried Wilhelm Leibniz – composition of the continuum – atomism

1. Introduction

Between December 1628 and August 1629, Pierre Gassendi undertook an extended tour of the Low Countries, which had a profound impact on the development of his Epicurean project. Having originally planned to compose an apology of Epicurus' life and ethical teaching, Gassendi left the Low Countries with the conviction that he would have to extend his study in order to cover the whole of Epicurean philosophy. The episode that contributed the most to this change of mind was probably his encounter in Dordrecht with Isaac Beeckman, whom Gassendi described in a letter as 'the best philosopher I have ever met'.[1] Also important was his sojourn at Louvain, where Gassendi was the guest of Erycius Puteanus, the author of an *Epicuri sententiae aliquot aculeatae ex Seneca* (1609), which he had recently read. At Louvain Gassendi also met some 'erudite humanists from Justus Lipsius' school': the professors of medicine Thomas Fienus and

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[1] See Gassendi's letter to Peiresc, 21 July 1629, in: P. Tamizey de Larroque, ed., *Lettres de Peiresc,* 7 vols, Paris, 1888-1898, vol. 4, p. 201.

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Petrus Castellanus, as well as Libertus Fromondus, who at the time was a professor of philosophy in the Arts Faculty.[2]

In 1631, two years after his friendly encounter with Gassendi, Fromondus published at Antwerp his *Labyrinthus sive de compositione continui liber unus,* in which he combated Epicurean atomism with both theoretical and religious arguments. Judging from the number of direct references to the book, it seems to have exerted less influence on Fromondus' contemporaries than his astronomical and cosmological works. In the correspondences of Galileo, Descartes and Mersenne, one easily finds allusions to the *Dissertatio de cometa* of 1619, to the *Meteorologicorum libri VI* of 1627, and to the two anti-Copernican tracts: the *Anti-Aristarchus* of 1631 and the *Vesta sive Aristarchus vindex* of 1634; however, there are only a few references to the *Labyrinthus.* It is mentioned in the draft of a letter written in 1632 by Gassendi to Gabriel Naudé. In the post-script, which he later cancelled, Gassendi briefly discussed Fromondus' anti-atomist arguments, together with those in Tommaso Campanella's *Atheismus triumphatus,* which was also published in 1631.[3] In a letter to Vopiscus Fortunatus Plempius, dated 3 October 1637, responding to some objections raised by Fromondus, Descartes praised the very subtle treatise *de compositione continui,* but without commenting on its content.[4]

For a clear echo of the *Labyrinthus*, one has to wait until the second half of the seventeenth century. As Daniel Garber has noted, Leibniz often 'adopted the figure of a labyrinth in connection with the continuum problem, and […] recommended the book as a valuable compendium of arguments and problems relating to the issue of the composition of the continuum'.[5] As far as I know, Leibniz alludes to the 'labyrinth of the continuum' for the first time in a letter to Henry Oldenburg of 11 March 1671; but the expression reappears several times both in his correspondence and in his published works.[6]

[2] F. Sassen, 'De reis van Pierre Gassendi in de Nederlanden (1628-1629)', *Mededelingen der Koninklijke Nederlandse Akademie van Wetenschappen, Afd. Letterkunde*, vol. 23, 1960, pp. 263-307.

[3] O.R. Bloch, *La philosophie de Gassendi. Nominalisme, matérialisme et métaphysique,* The Hague, 1971, p. 212.

[4] C. Adam and P. Tannery, eds, *Oeuvres de Descartes*, 13 vols, Paris, 1897-1913, vol. 1, p. 422. For the English translation, see J. Cottingham, R. Stoothoff, D. Murdoch and A. Kenny, eds and tr., *The Philosophical Writings of Descartes*, vol. 3, Cambridge, 1991, p. 65.

[5] D. Garber, 'Descartes, the Aristotelians, and the Revolution that Did not Happen in 1637', *Monist,* vol. 71, 1988, pp. 471-486 (473).

[6] Leibniz to Oldenburg, 11 March 1671, in: Gottfried Wilhelm Leibniz, *Sämtliche Schriften und Briefe,* ed. Preussische (later: Deutsche) Akademie der Wissenschaften zu Berlin, Berlin, 1923-, vol. 1, p. 90. See also Leibniz to Arnauld, 9 October 1687, in: Carl Immanuel Gerhardt, ed., *Die philosophische Schriften von Gottfried Wilhelm Leibniz,* 7 vols, Berlin, 1875-1890, vol. 2, p. 119; Gottfried Wilhelm Leibniz, *New Essays on Human Understanding,* P. Remnant and J. Bennett, eds and tr., Cambridge, 1981, II.xxiii.31.

In the secondary literature as well, the *Labyrinthus* has received less attention than other works by Fromondus. Only two studies have so far been devoted to this book. The first is a 1988 article by Geert Vanpaemel, entitled 'Libert Froidmont et l'atomisme', which examines the *Labyrinthus*' main anti-atomist arguments and Fromondus' subsequent critique of Descartes' matter theory.[7] The second is an insightful chapter in Philip Beeley's *Kontinuität und Mechanismus. Zur Philosophie des jungen Leibniz in ihrem ideengeschichtlichen Kontext*, which embeds Fromondus' thoughts on the continuum and his anti-atomist arguments in a detailed examination of the young Leibniz' evolving views on the structure of matter and motion.[8]

What has so far been missing is an examination of the *Labyrinthus* as a work in its own right. There are a number of reasons why this should be done. The *Labyrinthus* is interesting, first of all, from a historiographic point of view. For the intellectual historian, the early publication date of the book is puzzling. In 1631 Fromondus describes atomism as a theory which is spreading all over Europe, but it is not clear which authors and which books constitute the target of his critique. By that date, few works had been published in support of atomist ideas; moreover, several of the views that Fromondus subjects to criticism are known to us only from works that appeared after the *Labyrinthus*. Another reason for engaging with this book is the thoroughness of Fromondus' argumentation, as well as his original account of the history of atomism. As we shall see, the *Labyrinthus* presents the reader with surprising exegeses of ancient, medieval and early modern atomistic theories, and with a very interesting analysis of competing indivisibilist explanations of the composition of matter and motion. To make matters even more complex, Fromondus takes aim as often at Jesuit authors as he does at atomists, showing that the Aristotelian camp was as fragmented as the 'Epicurean' one. As we shall see, Fromondus endorses a nominalist theory of the composition of continuous magnitudes, which he tries to defend against the objections of Jesuit authors.

These three reasons for engaging with the *Labyrinthus* – its apparent historical prematurity, its systematic coherence and its originality – suggest the following structure for my treatment of the work. I shall first deal with the *pars destruens* of the *Labyrinthus,* in an attempt to shed light on Fromondus' possible targets, on his distinctive interpretation of the history of atomism, and on the

[7] G. Vanpaemel, 'Libert Froidmont et l'atomisme', in: A.-C. Bernès, ed., *Libert Froidmont et les résistances aux révolutions scientifiques. Actes du Colloque Château d'Oupeye, 26 et 27 septembre 1987*, Haccourt, 1988, pp. 131-143.

[8] P. Beeley, *Kontinuität und Mechanismus. Zur Philosophie des jungen Leibniz in ihrem ideengeschichtlichen Kontext*, Stuttgart, 1996, pp. 285-312.

arguments with which he combats the various forms of indivisibilism. Then, I shall compare the views criticised by Fromondus with those defended in a number of important atomist works published shortly after the *Labyrinthus*. Finally, I shall deal with the book's *pars construens*: Fromondus' nominalistic account of the composition of space, time and matter.

2. The *Labyrinthus* and its polemical targets

The stated goal of the *Labyrinthus,* a book consisting of 50 chapters and almost 200 pages, is to demonstrate that the atomist theory, which, according to Fromondus, had numerous followers at Louvain, was unfounded.[9] That atomism had found supporters in the Low Countries at the beginning of the seventeenth century is confirmed by the *Censurae opinionum* preserved in the *Archivium Romanum Societatis Iesu*. The first *censura* regarding the question of the composition of the continuum dates from 1606, when the Revisors General declared that the proposition 'the continuum is composed of a finite number of indivisibles', sent in from the *Provincia Belgica*, was 'erroneous in philosophy'.[10]

Thanks to the *Labyrinthus*, we know that an atomistic theory of the composition of the continuum was not only been taught in Belgian Jesuit schools, but also at the University of Louvain. Who exactly the Louvain atomists were, remains however unclear; for although Fromondus frequently refers to specific atomist positions held by some of his colleagues, he never mentions any of them by name.

In their controversy of 1634-1636, Fromondus and Gisbertus Voetius exchanged blows over the supposed 'licentiati' or 'doctores' who defended heterodox atomist theses at Louvain in violation of the verdict of the Council of Constance.[11] But under whom would these students have obtained their degrees? According to Garber, the most likely candidate is Erycius Puteanus, who in 1606 succeeded Justus Lipsius as professor of Latin.[12] Puteanus was in epistolary contact with Gassendi, to whom he sent a portrait of Epicurus, provided information about the most recent editions of Diogenes Laertius' *Vitae,* and with whom he discussed the identity of Epicurus' followers.[13] But Puteanus seemed to be

[9] Cf. Libertus Fromondus, *Labyrinthus sive de compositione continui liber unus,* Antwerp: B. Moretus, 1631, *ad Lectorem.*

[10] 'Continuum componitur ex indivisibilibus numero finito' (*Archivium Romanum Societatis Iesu*, F.G., 656A I, p. 319). See this and other *Censurae opinionum* in C.R. Palmerino, 'Two Jesuit Responses to Galilei's Science of Motion: Honoré Fabri and Pierre Le Cazre', in: M. Feingold, ed., *The New Science and Jesuit Science: Seventeenth Century Perspectives*, Dordrecht, 2004, pp. 187-227.

[11] See G. Monchamp, *Histoire du Cartésianisme en Belgique,* Brussels, 1886, p. 19.

[12] Garber, 'Descartes, the Aristotelians' (as in n. 5), p. 484, n. 16.

[13] See B. Rochot, *Les travaux de Gassendi sur Epicure et sur l'atomisme (1619-1658),* Paris, 1944, pp. 30-34.

interested mainly in Epicurus' ethics and, as far as I am aware, neither his printed works nor his numerous published letters show any trace of material atomism.

Another way to identify supporters of atomism is to search in the faculty of medicine. Unfortunately, due to the destruction of Louvain's archives in World War I, little is known about the theses defended in the early modern period.[14] The strongest indication that an atomist current might have existed in the faculty of medicine is the work of the physician Nicolaus Biesius (1516-1573), whose *De natura libri V* of 1573, reprinted in 1613, puts forward an atomist interpretation of Aristotle's theory of *minima naturalia*. Biesius, who ended his life as the personal physician to Emperor Maximilian II, was by no means a marginal figure in the Louvain establishment. But it is not clear, at least for the time being, whether he represented a tradition that continued until the days of Fromondus. To the best of my knowledge, no trace of atomism is found either in the works of Petrus Castellanus, *professor medicinae* and *linguae graecae*, or those of Thomas Fienus, *professor primarius* in medicine, whose *Dissertationes de cometa anni 1618* were published together with those of Fromondus.[15]

Finally, we should consider the faculty of arts, especially Lauren Ghiffene, who was the leading professor of philosophy. In his *Histoire du Cartésianisme en Belgique*, George Monchamp reports that, on 2 September 1626, François de Hinnisdael defended a number of theses formulated by Ghiffene, one of which dealt with the composition of the continuum. After mentioning that, according to some, continuous quantities were composed of atoms, whereas, according to others, they were composed of infinitely divisible parts, Ghiffene concluded: 'From this, bitter disputes arise. Is this surprising? They do not understand each other and therefore we shall teach freely'.[16]

The goal of the *Labyrinthus* is, in fact, to champion one of the two views mentioned by Ghiffene. On the basis of mathematical and physical arguments, Fromondus tries to prove that continuous magnitudes cannot be made out of indivisibles, but are instead divisible into infinitely divisible parts. It is worth

[14] This is the reason why Bruneel's *Répertoire* of medical theses, which should cover the period 1425-1797, only lists theses defended in the eighteenth century. See C. Bruneel, *Répertoire des thèses imprimées de l'Université de Louvain (1425-1797). Première partie. Faculté de Médecine. Fonds de la Bibliothèque centrale,* Louvain, 1977.

[15] *De cometa anni MDCXVIII dissertationes Thomae Fieni et Liberti Fromondi,* Antwerp: G.A. Tongris, 1619. Interestingly, the word 'atoms' (*atomi*) occurs in a passage of Fromondus' *dissertatio,* which explains why comets cannot be sublunary exhalations. Fromondus notices that the atoms composing the exhalation would ascend along divergent lines and that the exhalation would hence become too rarefied to form a comet (*De cometa,* p. 102).

[16] 'Hinc acerrimae disputationes: Quid mirum? se mutuo non intelligunt; nos ergo libenter docebimur', quoted in G. Monchamp, *Histoire* (as in n. 11), p. 14.

pointing out that Fromondus does not treat atomism as an homogeneous current, but rather distinguishes between the 'Epicureans' (*Epicureorum factio*), who postulate the existence of extended atoms, and those atomists who compose continuous magnitudes out of an infinite number of non-extended indivisibles. In the following, I shall use the terms 'physical atomism' and 'mathematical atomism' to designate the two theories criticised by Fromondus. By the term 'mathematical atomism' I do not mean 'a logical and geometrical speculation independent of the real physical explanations of natural phenomena',[17] but simply the view that atoms, as non-extended, are not only physically, but also mathematically, indivisible.

Although he rejects both physical and mathematical atomism, Fromondus directs his arguments mainly against the first theory, which he considers to be not only more dangerous from a religious point of view, but also conceptually more problematic. But before formulating his objections, Fromondus feels the need to set out the history of the theories he wants to combat.

3. The history of atomism according to Fromondus

The opening chapters of the *Labyrinthus* are devoted to showing that only a few Greek thinkers endorsed physical atomism, which was clearly incompatible with the principles of Euclidean geometry. In his attempt to limit the number of ancient atomists, Fromondus takes issue with the *Conimbricenses*, who ascribe to Pythagoras, Plato and Zeno of Citium the belief in the existence of extended atoms.[18] Aristotle's testimony in *Physics* 3.4 that 'Plato and Pythagoras posed the infinite in sensible things' is, in Fromondus' view, an obvious indication that these authors considered the parts of the sensible continuum to be indefinitely divisible.[19] As for Zeno of Citium, Fromondus quotes

[17] S. Gomez, 'From a Metaphysical to a Scientific Object: Mechanizing Light in Galilean Science', in: D. Garber and S. Roux, eds, *The Mechanization of Natural Philosophy,* Dordrecht, Heidelberg, New York and London, 2012, pp. 191-216 (194).

[18] Fromondus refers to the Commentary on the Physics, book 6, chapter 2, quaestio 2, where the *Conimbricenses* claim that 'affirmativam partem [= the composition of the continuum out of indivisibles] secuti sunt olim Stoici Philosophi, duce Zenone, itemque Pythagoras, Democritus, Leucippus' (*Commentarii Collegii Conimbricensis Societatis Jesu in octo libros Physicorum Aristotelis,* Lyon: Buysson, 1594, p. 232) and to the Commentary on De caelo, book 3, chapter 1, in which the *Conimbricenses* discuss Aristotle's critique of Plato's *Timaeus* (see *Commentarii Collegii Conimbricensis Societatis Iesu in quatuor libros De coelo Aristotelis Stagiritae,* Lyon: Giunta, 1594, pp. 401-402).

[19] 'Denique ipse Aristoteles alibi Pythagoricos & Platonem posuisse quoddam *infinitum in rebus sensibilibus,* id est, partes continui sensibilis, sine fine dividuas, diserte asserit', Fromondus, *Labyrinthus* (as in n. 9), p. 5. For an analysis of Fromondus' interpretation of the *Timaeus* see Beeley, *Kontinuität und Mechanismus* (as in n. 8), p. 291.

a number of sources which ascribe to Chrysippus the view that the division of matter can proceed *ad infinitum* and argues that all Stoics, including Zeno, must have been of the same opinion.[20] Fromondus criticises the *Conimbricenses* for mixing up Zeno of Citium and Zeno of Elea, and remarks that even the latter, whose paradoxes of motion are discussed by Aristotle in *Physics* 6.9, did not mean to argue for the composition of the continuum out of extended atoms, a view which is not reconcilable with Zeno of Elea's rejection of the void.[21]

The third chapter of the *Labyrinthus* is entitled: 'The more subtle among those who composed the continuum out of atoms made it out of infinite, not finite, atoms. Epicurus and Empedocles, however, preferred finite atoms'.[22] The 'more subtle atomists' are Leucippus and Democritus, who, according to Fromondus, were talented mathematicians and are therefore likely to have composed the continuum out of non-extended atoms. The only evidence he can adduce in support of this interpretation is the testimony of Diogenes Laertius, according to whom, Leucippus maintained that 'all things are infinite and change into one another', while Democritus believed that atoms were infinite in size and number.[23] There exists no consensus among scholars as to the nature of Democritus' atoms.[24] It is, however, interesting that Fromondus' opinion coincides with that of the German mathematician Georg Cantor (1845-1918), who also regarded Epicurus, and not Democritus, as the father of physical atomism:

> 'Thus we see that Leucippus, Democritus, and Aristotle consider the continuum a composite which consists *ex partibus sine fine divisibilibus*, but Epicurus and

[20] Fromondus' interpretation agrees with that of some contemporary scholars. See *e.g.* D.J. Furley, *Two Studies in the Greek Atomists,* Princeton, 1967, pp. 36-37.

[21] Fromondus, *Labyrinthus,* pp. 5-7. Fromondus is clearly thinking of the passage of the *Conimbricenses*' commentary on the Physics quoted in footnote 18. For Zeno's rejection of the void, see Diogenes Laertius, *Lives of Eminent Philosophers,* 9.29 (*Zeno of Elea*).

[22] 'Subtiliores inter eos qui continuum ex atomis struxerunt, ex infinitis potius quam finitis composuisse. Epicurum tamen et Empedoclem finitas maluisse', Fromondus, *Labyrinthus*, p. 9.

[23] *Ibid*., p. 10. Fromondus refers to Diogenes Laertius, *Lives of Eminent Philosophers,* 9.30 (*Leucippus*) and 9.44 (*Democritus*).

[24] Thomas Heath's opinion that Democritus was 'too good a mathematician' to believe in the indivisibility of geometrical magnitudes (T. Heath, *A History of Greek Mathematics,* vol. 1, Oxford, 1921, p. 181) was criticised by Furley, according to whom, 'Leucippus and Democritus [...] believed their atoms to be theoretically as well as physically indivisible' (see D.J. Furley, *Two Studies in the Greek Atomists* (as in n. 20), pp. 86, 100-101). Furley's interpretation, in turn, has been criticised as incompatible with Democritus' claim that atoms have different sizes and shapes (see P.S. Hasper, 'The Foundations of Presocratic Atomism', *Oxford Studies in Ancient Philosophy,* vol. 17, 1999, pp. 1-14 (4); A. Chalmers, *The Scientist's Atom and the Philosopher's Stone: How Science Succeeded and Philosophy Failed to Gain Knowledge of Atoms,* New York, 2009, pp. 19-42).

> Lucretius construct it out of their atoms considered as finite things. Out of this a great quarrel arose among the philosophers, of whom some followed Aristotle, others Epicurus; still others, in order to remain aloof from this quarrel, declared with Thomas Aquinas that the continuum consisted neither of infinitely many nor of a finite number of parts, but of absolutely no parts.'[25]

In the subsequent chapters Fromondus explains how Epicurean atomism, which in antiquity obviously represented a minority position, had a revival in the Middle Ages thanks to John Wycliff and to his pupil Jan Hus. But, as Fromondus is quick to point out, the eighth session of the Council of Constance in 1415, following the advice of *doctores et magistri* of the University of Oxford, placed an anathema on 45 *articuli principales* and 260 *articuli extrapendentes* of Wycliff. Among the latter was:

> 'Any continuous mathematical line is composed of two, three of four immediate points, or only of points which are simply finite. Or time is, was, or will be composed of contiguous instants. It is not possible that time and a line, if they exist, are composed in this way.'[26]

The first part was censured as 'a philosophical error', the second as 'an error concerning divine power'.[27] According to Fromondus, it was a matter of sheer historical contingency that this view was not definitively banned and forbidden. Pope John XXIII, who had presided over the eighth session of the Council, soon afterwards was forced to abdicate, and Martin V, who became the new Pope, confirmed the anathema on Wycliff's 45 principal articles, but not on the other 260, which included the article on the atomistic structure of space and time.

Although it was not condemned, atomism failed to find many followers in the Middle Ages. Fromondus believes that no one, apart from Wycliff and Hus,

[25] G. Cantor, *Grundlagen einer allgemeinen Mannigfaltigkeitslehre*, Leipzig, 1883, in: W. Ewald, *From Kant to Hilbert, A Source Book in the Foundations of Mathematics,* 2 vols, Oxford, 1999, vol. 2, p. 903.

[26] 'Quinquagesimus [articulus] est iste: *Linea aliqua Mathematica continua componitur ex duobus, tribus, vel quatuor punctis immediatis, aut solum ex punctis simpliciter finitis. Vel tempus est, fuit, vel erit compositum ex instantibus immediatis. Item non est possibile, quin tempus & linea, si sint, taliter componantur*', Fromondus, *Labyrinthus,* p. 12. Fromondus' source might be Peter Crabbe & Gratius Ortuinus, *Concilia omnia, tam generalia quam particularia: ab apostolorum temporibus in hunc vsque diem a sanctissimis patribus celebrata […] tomus secundus,* Cologne, 1538, *Appendix: Errores Ioannis Wicleff de Anglia & Ioannis Husz de Bohemia damnati in hoc Sacro generali Constantiniensi concilio.*

[27] 'Et subditur proxime censura: *Prima pars est error in Philosophia: sed ultima errat circa divinam potentiam*', Fromondus, *Labyrinthus,* p. 12.

asserted the composition of the continuum out of extended indivisibles, since neither the Church Fathers, nor the three main philosophical schools – the Thomists, Scotists, and Ockhamists – wished to deviate from Aristotle in this matter. The only partial exception was John Maior in the first half of the sixteenth century, who, however, composed the continuum out of infinite points.[28] Thanks to the research of John Murdoch and other scholars who followed in his footsteps, we now know that Fromondus' list of medieval atomists is incomplete, and that it should also include Gerard of Odo, Nicholas Bonet, Nicholas of Autrecourt, Henry of Harclay, William Crathorn and Walter Chatton, some of whom believed continuous magnitudes to be composed of a finite number of indivisibles, whereas others asserted the existence of infinite indivisibles.[29] Having attempted, by means of his unconventional and deliberately skewed interpretations of ancient and medieval sources, to convince his readers that Epicurus' matter theory was endorsed by only a few thinkers and rejected by most, Fromondus moves on to summarise the mathematical and physical arguments which were put forward by Aristotle and other anti-atomist authors.

4. Fromondus' anti-atomistic arguments

Fromondus' strategy in the long *pars destruens* of the *Labyrinthus* is to demonstrate that physical atomism is not only incompatible with the principles of Euclidean geometry (chapters 8-15), but also unable to account for a great number of physical phenomena (chapters 16-30). Since he shares Aristotle's belief in the isomorphism of all physical magnitudes, Fromondus starts by discussing the composition of the permanent continuum, that is, of physical extension, and then applies his conclusions to time and motion.[30]

[28] *Ibid.,* p. 25. For Maior's theory of the composition of the continuum, see Joannes Maior, *Editio secunda in secundum librum sententiarum nunquam antea impressa*, Paris: Granjon, 1519, *distinctio II, quaestio I*. The last sentence of this *quaestio* ('Valeat hoc paradoxum quantum valere potest'), which incidentally was not contained in the first edition (1510), is interpreted by Fromondus as a sign that Maior was hesitant about the issue of the composition of the continuum.

[29] On medieval atomism see, among others, J. Murdoch, 'Infinity and Continuity', in: N. Kretzman, A. Kenny, J. Pinborg, eds, *The Cambridge History of Later Medieval Philosophy,* Cambridge, 1982; Id., 'Atomism and Motion in the Fourteenth Century', in: E. Mendelsohn, ed., *Tranformation and Tradition in the Sciences. Essays in Honor of I.B. Cohen,* Cambridge, 1984, pp. 45-66; N. Kretzmann, ed., *Infinity and Continuity in Ancient and Medieval Thought,* Ithaca and London, 1982; C. Grellard and A. Robert, eds, *Atomism and Its Place in Late Medieval Philosophy*, Leiden, 2009.

[30] Fromondus, *Labyrinthus,* p. 22. On Fromondus' belief in the isomorphism of space, time and motion see Beeley, *Kontinuität und Mechanismus*, p. 304.

Always keen to emphasise the orthodoxy of his own position, Fromondus lists the sources from which he derives his anti-atomist arguments. The names we encounter most frequently are those of Thomas Aquinas, Duns Scotus and Gregory of Rimini.

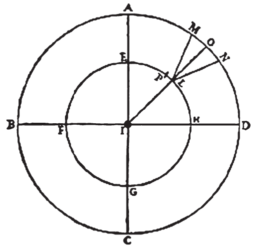

Figure 1: Epicurean atomism does not allow to account for the one-to-one correspondence between the points of two concentric circumferences

It is from Duns Scotus that Fromondus borrows the first of his geometrical arguments: If the two concentric circumferences of figure 1 were composed of a finite number of points, then a point L on the smaller circumference would correspond to two points M and N on the larger circumference, which violates the principle that 'each of the straight lines drawn from a point of the larger circumference to the centre must necessarily pass through a different point of the smaller circumference'.[31] Fromondus goes on to argue that, on the assumption that lines are composed of a finite number of extended points, it is impossible to account not only for the one-to-one correspondence between the points composing lines of different lengths, but also for the existence of incommensurable magnitudes such as the circumference and the diameter of a circle, or the side and the diagonal of a square. Moreover, physical atomism does not allow for the very existence of some specific geometrical figures, as it is impossible to construct a circle or an isosceles triangle out of extended points. As Geert Vanpaemel has observed, in order to support this claim, Fromondus 'transforms space into a regular grid of points', in which one can only move horizontally or vertically

[31] 'Itaque inprimis docent Mathematici, quod [...] omnes lineae rectae, ductae a quolibet puncto circumferentiae maioris ad centrum, debant necessario transire per aliud & aliud semper circumferentiae minoris punctum, & numquam duae per idem', Fromondus, *Labyrinthus,* p. 30. The argument, as well as the figure, is borrowed from Johannes Duns Scotus, *Quaestiones in librum secundum sententiarum, distinctio II, quaestio IX*, in: *Opera Omnia,* ed. L. Wadding, vol. 6, Lyon: Durand, 1636 (reprint Hildesheim, 1968-1969), pp. 230-233. In the modern edition, which does not contain illustrations, this *quaestio* corresponds to *Ordinatio secunda, distinctio II, pars II, quaestio V* (*Doctoris subtilis et mariani Ioannis Duns Scoti [...] Opera Omnia,* vol. 7, Città del Vaticano: Typis Polyglottis Vaticanis, 1973, pp. 290-298).

between two points.[32] By means of figures 2 and 3, below, Fromondus shows how a circle becomes a square and how the diagonal of a square is transformed into an indented line.

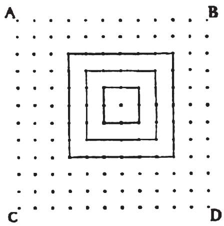

Figure 2: The circle becomes a square

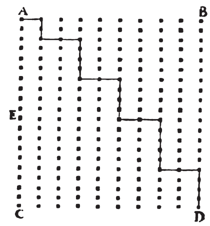

Figure 3: The diagonal of a circle becomes an indented line

The last two geometrical arguments are taken from the field of optics, and are meant to show that light cannot have a discrete structure. Fromondus argues, for example, that if the sun were made out of a finite number of points, it could only emit a finite number of light rays. But if this were the case, the increasing divergence of the rays over space would cause the world to be illuminated only intermittently.[33]

Although Fromondus does not mention any seventeenth-century atomists by name, we know that some of his contemporaries would not have found his objections compelling. Take, for example, Sébastien Basson, who in his *Philosophia naturalis* of 1621 argued that the geometrical objections against atomism stem from a wrong view about the nature of geometrical entities. Basson addressed the traditional objection, which is also formulated in the *Labyrinthus*, that the composition of lines out of extended points would not account for the incommensurability between the side and the diagonal of a square. His answer was that 'the nature of the individual [i.e., point or atom] does not allow the first lines to be truly straight, apart from those that are parallel and those that cross at a perfect right angle'.[34] This means, in other words, that the diagonal of a square is

[32] 'Fromondus transforme l'espace en un treillis régulier de points', Vanpaemel, 'Libert Froidmont et l'atomisme' (as in n. 7), p. 133.

[33] For Fromondus' seven geometrical arguments against physical atomism see *Labyrinthus,* pp. 29-56.

[34] 'Natura enim individui, ut probat quae de punctorum contactu docuimus, non patitur primas lineas vere rectas dari praeter quam parallelas; et perfecte transversas', Sébastien Basson, *Philosophiae naturalis adversus Aristotelem libri XII*, Geneva, 1621, p. 417. On Basson's atomism, see C. Lüthy, 'Thoughts and Circumstances of Sébastien Basson. Analysis, Micro-History, Questions', *Early Science and Medicine,* vol. 2, 1997, pp. 1-73.

not a straight line, but an indented one, which is precisely the view ridiculed by Fromondus' in the *Labyrinthus*.

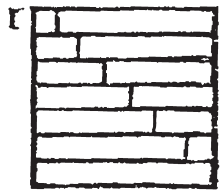

Figure 4: The diagonal of a square according to Basson

In his *Philosophia*, Basson went so far as to deny that God could create two straight lines and make them cross at any angle he wished.[35] Another way to answer the geometrical objections against atomism consisted in introducing a distinction between physical and mathematical divisibility. This strategy was to be adopted, for example, by Jean Chrysostome Magnen, who in his *Democritus Reviviscens* (1646) asserted that space, time and motion are divisible ad infinitum 'extrinsically, or mathematically, but not physically' (*extrinsece et mathematice, non autem physice*).[36]

Having shown to his own satisfaction that the composition of the continuum out of extended indivisibles is incompatible with Euclidean geometry, Fromondus sets out to prove that the atomists' hope to achieve a victory over 'Aristotle and all the mathematicians' in the field of physics is completely unfounded.[37] Chapters 16-30 of the *Labyrinthus* are devoted to showing that physical atomism fails to provide a convincing explanation of a whole range of natural phenomena, most notably the acceleration and deceleration of motion and the condensation and rarefaction of matter.

Fromondus' first physical argument, which is a variation on Zeno of Elea's paradox of Achilles and the tortoise, holds that if space and time were made up of extended indivisibles, a winged horse would not be able to overtake a tortoise which started the race with a slight advantage. According to Epicurus' theory, both animals would move at the constant speed of one minimum of space per minimum of time. Fromondus explains that atomists have only two ways of escaping from this difficulty, both of which he finds unsatisfactory. The first

[35] S. Basson, *Philosophiae naturalis* (as in n. 34), p. 422.

[36] Jean Chrysostome Magnen, *Democritus reviviscens, sive de atomis*, Pavia: J.A. Magrius, 1646, p. 97.

[37] The nine physical arguments against atomism are given in Fromondus, *Labyrinthus*, pp. 56-97.

consists in assuming that the flying horse traverses several points of space in one moment of time. This solution, which Fromondus does not ascribe to any particular author, is untenable because it implies that the horse would occupy several *loci adequati* in the same indivisible instant of time. The other way to explain the fact that two bodies can traverse the same distance at different speeds is to assume that the slower motion is periodically interrupted by moments of rest, a solution which, he says, is adopted by 'many at Louvain and a few in Spain'.[38] According to Fromondus, such an explanation not only contradicts sensory experience, which shows that even the motion of very slow bodies is continuous, but is also absurd from a theoretical point of view, since it implies that the *causa prima,* that is, God, periodically intervenes to interrupt the action of the *causae moventes*.[39]

The only Spanish source that Fromondus cites in this chapter is Franciscus Vallesius' *Controversiarum medicarum et philosophicarum libri decem,* first published in 1556. In book 3, chapter 8 of this work, which is devoted to the measurement of the pulse, there is a long digression on the velocity and slowness of motion. Vallesius ascribes to Galen a theory which he considers analogous to that of Zeno and the Stoics, which states that a body moves more slowly than another if it rests longer in each point of space.[40] As we shall see below, the view criticised by Fromondus was defended the following year by Roderigo de Arriaga in his *Cursus Philosophicus* of 1632.

Fromondus also refers to his encounter with a 'not unknown English philosopher' ('philosophus non exigui nominis Anglus') who had become an Epicurean during his studies at Salamanca. When asked why the motion of a falling stone should be interrupted by moments of rest, this philosopher, who, as Vanpaemel has suggested, was probably Kenelm Digby, gave the ridiculous answer that the pauses were necessary for the stone to regain force.[41]

The six subsequent arguments are merely adaptations of the previously formulated geometrical arguments, for, as Fromondus himself acknowledges, mathematical arguments become physical when applied to sensible matter. The fact that no one-to-one correspondence can be established between the extended

[38] 'Plures hic Lovanij, & pauci quidam in Hispania, ut iam intelligo, ab Aristotele ad Epicurum transfugae, sistere testudinem', *ibid.,* p. 62.

[39] *Ibid*., pp. 63-66.

[40] 'Hic locus continent opinionem Galeni de latione, quae Zenonis fuit, & Stoicorum multo ante', Franciscus Vallesius, *Controversiarum medicarum et philosophicarum libri decem*, Hanau: C. Marinus, 1606, pp. 132-135. Vallesius, who does not cite his source, is likely to have mixed up Zeno of Elea and Zeno of Citium.

[41] Fromondus, *Labyrinthus,* pp. 64-65. See Vanpaemel, 'Libert Froidmont et l'atomisme', p. 133.

indivisibles of two concentric circumferences is now used to show that the spokes of a turning wheel would break, that an iron hoop moved in circle by a whip would remain at rest for a very long time, that a sundial would not be able to project its shadow on the part of a plane opposite to the sun, that no accurate image of a large body could be formed on the pupil of the eye, and that it would be impossible to build a tower perpendicular to the ground.[42] In addition, Fromondus uses the proof that in an atomistic space the diagonal of a square becomes an indented line to argue that no river could be navigated: the inclination of a long river such as the Nile would be such that it would end up in the centre of the earth.[43]

The final physical arguments of the *Labyrinthus* deal with the puzzling phenomenon of the rarefaction and condensation of matter. Fromondus explains that, properly speaking, 'condensation' is a state in which a body 'occupies less space under the same quantity of parts'.[44] According to their more common meanings, however, the terms 'rarefaction' and 'condensation' are used to describe the behaviour of porous bodies which, like a sponge, expand when particles of air or of another material enter into their pores and contract when these particles are extruded. Fromondus says that many modern 'Epicureans', as well as some Peripatetics, only admit rarefaction and condensation in the commonly accepted sense.[45]

Those Epicureans, however, who admit the occurrence of rarefaction and condensation in the proper sense of these terms are faced with a choice between two possible explanations: either they postulate the presence of interstitial voids within rarefied bodies and the mutual interpenetration of particles in condensed bodies, or they assume that the atoms composing a body multiply during rarefaction and contract during condensation. Unsurprisingly, neither of these solutions satisfies Fromondus.

His refutation of the vacuum hypothesis is explicitly directed against the theory put forward by Adrien Turnèbe in the *Libellus de calore* of 1600, in which the expansion of light and the diffusion of heat are adduced as evidence for the presence of tiny voids in the air. Turnèbe argues that, in addition to these

[42] Fromondus, *Labyrinthus*, pp. 67-76.

[43] *Ibid.,* pp. 73-74.

[44] 'Propriam [condensationem] voco [...] per quam corpus sub eadem partium multitudine minus spatium occupat', *ibid.,* p. 81.

[45] Fromondus refers specifically to Otto Casmann, who in the sixth question of his *Problemata Marina* (1546) explains rarefaction as the result of the entrance of air particles through the pores of bodies. According to Fromondus, Casman's explanation is in contradiction to easily observable phenomena such as the fact that a sealed bottle, completely filled with water, explodes upon freezing.

interspersed voids, there exists in nature 'another certain emptiness, immense and extending widely, which nature avoids', but which, as Hero of Alexandria convincingly proved, can nonetheless be brought about:

> 'If you would pierce through a glass ball and you would insert a small siphon through the hole and you would carefully caulk around the hole, with your mouth you could draw out the air which is inside and there would remain a great emptiness within. The proof of this would be the following: if before the air is let in from the outside, you were to cover the siphon with your finger and you were to turn it upside down in water, the liquid would enter when you would remove your finger; and it would be drawn into the interior of the ball.'[46]

Fromondus accepts the result of this experiment, but denies that a vacuum is created in the glass ball. When the ball is immersed in water, the highly rarefied air left in it suddenly condenses, which causes a concomitant ascent of water in the ball.[47] As a good Aristotelian, Fromondus resolutely denies the possibility of a vacuum and argues that all hydraulic machines and artificial fountains function according to the principle of the *horror vacui*.

He explains that, as well as the void, there is another major 'natural hatred': the interpenetration of parts. Did Aristotle not deny the substantial nature of light precisely because of its ability to penetrate air? Book 2, chapter 7 of *De anima* shows that 'two bodies cannot be in the same place at the same time', which is why, in later times, Augustine, Gregory of Rimini, Thomas Aquinas, Duns Scotus and others concluded that the penetration of matter could only take place 'Dei virtute supernaturali'.[48] Some atomists tried to avoid this difficulty by arguing that the interpenetration of matter takes place not by means of local motion, but through an alteration of sorts (e.g. refrigeration), an explanation that Fromondus finds unconvincing, because he does not see how an alteration can be brought about by any other cause than local motion or a divine miracle.[49]

Having shown in chapter 29 that condensation cannot be produced by the interpenetration of matter, Fromondus devotes chapter 30 to criticizing those Epicureans who believe that 'individual points are replicated in a rarefied body'

[46] Adrien Turnèbe, *Libelli de vino, calore et method,* Paris: Claudius Morellus, 1600, fols 15v-16r, quoted in: C.B. Schmitt, 'Experimental Evidence for and against a Void: The Sixteenth-Century Arguments', *Isis*, vol. 58, 1967, pp. 352-366 (361).

[47] 'Sane faciet; sed non quia vacuum in pila vitrea, sed aër admodum rarefactus fuit, qui frigore aquae subito inhorrescit & se contrahit, aquamque metu vacui sursum invitat', Fromondus, *Labyrinthus,* p. 85.

[48] *Ibid.*, p. 89.

[49] *Ibid.,* p. 94.

and that 'condensation is the diminution of that replication'.[50] According to this theory, a condensed body has the maximum density which can be obtained naturally and without penetration of parts.

Fromondus mentions a number of reasons why he considers this explanation even more improbable and absurd than that which postulates the formation of void spaces and the interpenetration of matter. Firstly, the replication of a body's particles would imply that one atom of matter is at the same time present in many points of space, which is obviously impossible; secondly, if all particles of a body multiply at the same time, penetration must necessarily occur; finally, rarefaction is often accompanied by a remission of a body's colour, which would not be the case if it were due to a multiplication of the material particles.[51]

As we have seen, Fromondus' critical arguments in the *Labyrinthus* are directed especially against Epicurean atomism, which postulates the composition of physical magnitudes out of extended indivisibles. Before describing his own way out of the labyrinth of the continuum, Fromondus devotes a chapter to those authors 'who try in vain to find a middle way between Aristotle and Epicurus, either by denying the existence of any parts in the continuum, or by affirming that those parts are infinite and indivisible'.[52] The first view, that 'the continuum is an entity from which parts can be separated, but in which no parts are contained before they are separated', is rejected by Fromondus because separating from a continuum parts which are not formally contained in it is the same as extracting money from an empty purse.[53] Despite what Fromondus says, this theory, far from being a compromise between the positions of Aristotle and Epicurus, is, in fact, fully compatible with the explanation of the speed and slowness of motion in *Physics* 6.2, where we read that 'the continuity of time follows on that of magnitude and also the continuity of magnitude on that of time, for divisions and subdivisions of the given time and the given magnitude can always be made to keep pace in number and in ratio without limit'.[54] Sander De Boer

[50] 'Caput XXX: 'Nec condensatio explicari potest per diminutionem replicationis, qua singula puncta replicata esse aiunt in corpore raro', *ibid.,* p. 95.

[51] *Ibid.,* p. 96.

[52] 'Caput XXXI: Frustra quidam conati inter Aristotelem & Epicurum medij incedere, negando ullas esse in continuo partes, aut asserendo infinitas, sed indivisibiles', *ibid.,* p. 97.

[53] 'Continuum enim, aiunt, est tale ens unde exscindi partes possunt, sed in quo tamen, priusquam exscindas, non continentur […]. Sed in continuo tamen profecto res habet aliter: in quo nisi partes formatae & formaliter continentur, non magis inde eximi poterunt, quam pecunia ex crumena inani', *ibid.,* p. 97.

[54] Aristotle, *Physics* 6.2 (233a12-17). The passage quoted is taken from Aristotle, *Physics,* P.H. Wicksteed and F.M. Cornford, tr., 2 vols, Loeb Classical Library, Cambridge, Mass., London, 1934, p. 109.

has observed that Aristotle's theory 'takes the whole existing continuum as its starting point, and therefore never implies a primacy of the indivisible parts over the whole'.[55] As we shall see, contrary to Aristotle, Fromondus champions a theory of the composition of the continuum which safeguards the primacy of the parts over the whole. In the concluding chapters of the *Labyrinthus*, he argues that continuous magnitudes are not only infinitely divisible, but also actually composed of an infinite number of ever decreasing parts.

The second theory, which in chapter 3 Fromondus had attributed to Leucippus, Democritus and Johannes Maior, holds that continuous magnitudes are composed of an infinite number of non-extended indivisibles. He admits that, compared to physical atomism, mathematical atomism has the advantage of being in accordance with the principles of Euclidean geometry. From a physical point of view, however, it is difficult to understand how non-extended indivisibles could be the constituents of continuous magnitudes. If space and time were composed of an infinite number of successive non-extended atoms, no interval of time could elapse and no portion of space could be traversed. Moreover, if bodies were composed of such atoms, they would be devoid of gravity, as Aristotle demonstrates in book 3, chapter 1 of *De caelo*.[56]

Chapter 31 marks the end of the long *pars destruens* of the *Labyrinthus*. In the remaining part of the work (chapters 32-50), Fromondus sets out to prove that no positive indivisibles are found in the continuum, but only infinitely divisible parts. Before, however, analysing Fromondus' own theory as to the composition of continuous magnitudes, I wish to dwell on a very interesting aspect of his critique of atomism: the emphasis he puts on the isomorphism of space, time and matter.

5. The isomorphism of space, time and matter according to Fromondus and his atomist contemporaries

In Fromondus' physical arguments, we can observe an interesting symmetry between his critique of atomistic explanations of the acceleration and deceleration of motion, on the one hand, and of the rarefaction and condensation of matter, on the other. In both cases, Fromondus notes that Epicureans face the choice between an explanation based on the concept of 'discontinuation' and one based on the concept of 'replication'.

[55] S. De Boer, 'The Importance of Atomism in the Philosophy of Gerard of Odo (O.F.M.)', in: C. Grellard and A. Robert, eds, *Atomism and Its Place in Late Medieval Philosophy* (as in n. 29), pp. 85-106 (89).

[56] Fromondus, *Labyrinthus,* pp. 98-99.

In the previous section we have seen Fromondus criticize those atomists who, in order to account for variations of speed, assume that 'in every slow motion, some pauses and retardations occur in which the moving object rests, but which in a faster movement are filled'.[57] The view that the rarefaction of matter must be explained as the result of the intermixture of tiny voids among the atoms seems equally unconvincing to him. That Fromondus is aware of the similarity between these two 'discontinuist' explanations appears from the fact that he uses analogous arguments to criticize them. In chapter 18 he argues that if the only continuous motion in nature were that of the prime mover, the motions of terrestrial bodies would contain many more atoms of rest than of motion. A similar observation is found in chapter 28, where we read that bodies such as gun powder and aquavit, which can rarefy up to 125,000 times, in their rarefied state would contain more *vacuola* than material particles**.**

The other explanation discussed by Fromondus consists in assuming that indivisibles of matter and motion can replicate themselves. According to some atomists, variations in speed occur because a body can traverse more or fewer indivisible units of space in one instant of time, whereas rarefaction is made possible by the fact that an atom of matter can expand to occupy more than one indivisible unit of space. The similarity between the two explanatory patterns is in this case even more obvious than in the previous one. When arguing that rarefaction cannot take place 'per replicationem', Fromondus refers back to the arguments used in chapter 17, where he has proven that acceleration cannot be due to the 'replication of motion', for a body cannot be in several places at the same time.[58]

As has previously been observed, it is difficult to identify the targets of Fromondus' critique, since he is more eager to mention his friends than his foes. Although it is evident that the theories criticized in the *Labyrinthus* had been circulating in Europe for some time, they are only known to us thanks to works published shortly after 1631. What these works have in common with the *Labyrinthus* is that they explicitly assert the isomorphism of space, time and matter and, as a consequence, establish a connection between the phenomenon of the rarefaction and condensation of matter, on the one hand, and of the acceleration and deceleration of motion, on the other.[59]

[57] 'In omni motu tardo pausas et morulas quasdam interiiciunt quibus mobile quiescat, quae in motu celeriori complentur', *ibid.*, p. 62.

[58] *Ibid.,* p. 95.

[59] For a more in-depth analysis of the works discussed in this section, see C.R. Palmerino, 'The Isomorphism of Space, Time and Matter in Seventeenth-century Natural Philosophy', *Early Science and Medicine*, vol. 16 (2011), pp. 296-330.

A very clear example of the 'discontinuation' thesis, which is extensively criticized by Fromondus, is found in the *Cursus philosophicus* by the Spanish Jesuit Roderigo de Arriaga, the first edition of which was published one year after the *Labyrinthus*. In the sixteenth *disputatio physica*, which is devoted to the issue of the composition of the continuum, an explicit link is established between the phenomenon of the rarefaction and condensation of matter, which is the object of section X, and that of the acceleration and deceleration of motion, which is discussed in section XI. In section X Arriaga explains the rarefaction and condensation of matter as the result of the introduction or expulsion of particles of air (and not of void). This seems to confirm Fromondus' view that many modern Epicureans only admit rarefaction in the vulgar meaning of this term (*condensatio vulgaris*). [60] As far as the acceleration and deceleration of motion is concerned, Arriaga maintains, in section XI, that the variations of speed are due to the fact that God periodically interrupts the action of the *causae moventes*. According to him, the only continuous motion in nature is that of the first moveable, which takes place at the speed of one minimum of space per minimum of time.[61] As we have seen, the hypothesis that God had to intervene in order to restore a body's motion after each pause appeared ludicrous to Fromondus. A few years later Pierre Gassendi was to propose a more sophisticated explanation of the discontinuity of motion. In his *Syntagma Philosophicum,* published posthumously in 1658, Gassendi maintained that the alternation of motion and rest in the motion of macroscopic bodies was the result of clashes between the composing atoms.[62]

As Fromondus pointed out, those atomists who did not endorse the discontinuity thesis, attributed to the indivisibles of space, time and matter the property

[60] 'Dicendum ergo est […] rarefactionem aquae fieri per introductionem aliquorum corpusculorum aëris, aut aliorum […] ratione autem illorum maiorem occupari locum a corpore raro quam antea; in condensatione vero foras expelli eiusmodi corpuscula, ideoque minorem locum occupare. Haec sententia clarissime explicat maiorem vel minorem rarefactionem, quin aliqua corpora penetrentur aut relinquant vacuum', Roderigo de Arriaga, *Cursus philosophicus,* Antwerp: B. Moretus, 1632, p. 428. Arriaga's explanation of rarefaction and condensation resembles that of Otto Casman criticised by Fromondus in the *Labyrinthus* (see above, n. 45).

[61] *Ibid.*, p. 432. O.R. Bloch notes that Arriaga's theory was also mentioned by Mersenne in a passage of his *Harmonie universelle*: 'L'esprit humain n'est pas capable de comprendre comme il est possible qu'un mouvement continu soit plus tardif qu'un autre: ce qui a contraint le Philosophe Hespagnol Arriaga dans sa seiziesme dispute physique, et plusieurs d'autres, de dire que la tardiveté du mouvement n'est autre chose qu'une interruption de plusieurs repos', Marin Mersenne, *Harmonie universelle contenant la theorie et la pratique de la musique*, 2 vols, Paris: S. Cramoisy, 1636, vol. 1, p. 74, quoted in Bloch, *La philosophie* (as in n. 3), p. 226, n. 109. For the medieval background to the hypothesis of the discontinuity of motion, see Murdoch, 'Atomism and Motion' (as in n. 29).

[62] See Pierre Gassendi, *Syntagma Philosophicum,* in his *Opera omnia in sex tomos divisa,* Lyon: Anisson & Devenet, 1658, vol. 1, p. 341. For Gassendi's account of rarefaction and condensation, see *ibid.*, p. 194, A-B.

of 'replicating' themselves. This was, for example, the case of Jean Chrysostome Magnen, whose influential *Democritus reviviscens* was published 15 years after the *Labyrinthus*. In the last *disputatio* of his book, Magnen recognizes that the acceleration and deceleration of motion and the rarefaction and condensation of matter are extremely difficult for an atomist to deal with. As far as the first phenomenon is concerned, Magnen observes that no microscopic pauses are needed in order to account for the variations of speed of moving bodies. The acceleration of fall can, for example, be explained by the fact that each atom of the falling body acquires a new atom of impetus in each successive atom of time.

Not only the acceleration of motion, but also the rarefaction of matter is interpreted by Magnen as the result of a replication of sorts. In chapter 2, *De rarefactione et condensatione iuxta Democritum,* he explains that 'a single atom, without rarefaction, inflation, or reproduction, can naturally occupy a bigger and bigger place *ad infinitum*', simply by changing its shape. For as is well known, solids of equal volume but different shapes also have different external surfaces.[63]

A similar theory was espoused by the French Jesuit Honoré Fabri, who, like Magnen, postulated the existence of indivisibles of changing size. In the ninth book of his *Metaphysica demonstrativa,* a collection of lectures published in 1648 by his pupil Pierre Mousnier, Fabri argues that if space and time were composed of equal indivisibles, it would be impossible for a body to traverse more or less than one indivisible of space in each successive instant of time.[64] Fabri's own explanation of the variety of speeds of physical bodies is based on the following principle: although a moving body can only pass through one *locus adaequatus* in each physical instant of time, successive instants can each have a different duration. For while 'there can be nothing smaller than a mathematical instant, things are different in the case of the physical instant, which is potentially divisible'.[65] Interestingly, Fabri's explanation circumvents an objection formulated by Fromondus in the *Labyrinthus*: that it is impossible for an

[63] 'Atomus est omnis figurae capax, ergo occupare potest maiorem, & maiorem locum in infinitum: cum enim figurae regulares in isoperimetris sint magis collectae minoremque locum occupent, sequitur quod quo irregularior erit figura eo maiorem occupabit locum, at non potest dari, ita irregularis, quin magis irregularis esse possit, ergo etiam maioris loci capax', Jean Chrysostome Magnen, *Democritus reviviscens, sive de atomis*, Pavia: J.A. Magrius, 1646, p. 247.

[64] *Metaphysica demonstrativa, sive scientia rationum universalium, auctore Petro Mousnerio doctore medico. Cuncta excerpta ex praelectionibus R.P. Honorati Fabri,* Lyon: I. Champion, 1648, pp. 369-383.

[65] 'Nam equidem fateor instanti mathematico nihil esse posse minus; secus vero instanti physico, quod est divisibile potentia, ut dicemus alias', *Tractatus physicus de motu locali, in quo effectus omnes, qui ad impetum, motum naturalem, violentum, et mixtum pertinent, explicantur. Auctore P. Mousnerio cuncta excerpta ex praelectionibus R.P. Honorati Fabri,* Lyon: I. Champion, 1646, p. 110.

indivisible unit of matter to occupy several *loci adequati* in the same indivisible instant of time.

Like Arriaga and Magnen, Fabri establishes an explicit link between his theory of matter and his theory of motion. He observes, in fact, that the hypothesis of the actual indivisibility, but potential divisibility, of physical points 'makes it easy to explain all phenomena related to quantity: first, the speed and slowness of motion […]; second, rarefaction and condensation, compression and dilatation; for every point can have a bigger or smaller extension'.[66] According to Fabri, material bodies are composed of physical points of different shapes that under certain conditions become bigger or smaller, thereby causing rarefaction or condensation.[67]

Although the theories discussed in this section bear a strong resemblance to those set out in the *Labyrinthus,* there is no evidence that the book was actually read by either Arriaga, Magnen or Fabri. As for Gassendi, his only reference to the *Labyrinthus* is found in the letter to Gabriel Naudé that I mentioned in the introduction.

An author who, by contrast, might have been directly influenced by Fromondus is Galileo Galilei. In his last work, the *Two New Sciences*, published in 1638, Galileo abandoned his previous physical atomism in favour of mathematical atomism. He now maintained that a material body was composed of an infinite number of non-extended atoms, a finite distance of an infinite number of points, a finite time-interval of an infinite number of instants without duration, and an accelerated motion of an infinite number of degrees of speed.[68] Galileo was confident that his matter theory would be acceptable to his Aristotelian colleagues:

> 'But by employing the method I propose […] I believe that they [i.e., the learned Peripatetics] should be satisfied, and should allow this composition of the continuum out of absolutely indivisible atoms. Especially since this is a road that is perhaps more direct than any other in extricating ourselves from many intricate *labyrinths* […]. One such is the understanding of rarefaction and condensation, without stumbling into the inconsistency of being forced by the former to admit void spaces, and by the latter to admit the interpenetration of bodies.'[69]

Pietro Redondi was the first to point out that these lines may conceal a reference to the *Labyrinthus,* in which – as we have seen – mathematical atomism was

[66] 'Facile iuxta hanc hypothesim, omnia quae pertinent ad quantitatem explicantur; Primo motus velocitas et tarditas […]. Secundo rarefactio, condensatio, compressio, dilatatio; quia quodlibet punctum potest habere, modo maiorem, modo minorem extensionem', Fabri, *Metaphysica* (as in n. 64), p. 414.

[67] *Ibid.,* p. 395.

[68] See Palmerino, 'The Isomorphism' (as in n. 59), pp. 19-23.

[69] Galileo Galilei, *Two New Sciences,* S. Drake, ed. and tr., Madison, 1974, p. 54.

described as 'more subtle' than physical atomism.[70] As is well known, Redondi believes that the chief reason for Galileo's condemnation in 1633 was not his endorsement of heliocentrism in the *Dialogue* of 1632, but rather his commitment to physical atomism in the *Assayer* of 1623.[71] The conception of matter as a composite of extended atoms was perceived by the Church authorities as 'heretical', since it made it impossible to account for the presence of the body and blood of Christ under the accidents of the eucharistic bread and wine. This would explain why, in his final work, Galileo abandoned his previous corpuscular physics in favour of a more 'prudent' matter theory.[72] As mentioned above, in the fourth chapter of his *Labyrinthus* Fromondus reports that Wyclif's atomism was condemned at the Council of Constance. The justification given by the Church authorities at the time – that 'the composition of the continuum out of infinite parts does not imply a contradiction, and hence cannot be denied without infringing God's omnipotence' – would obviously not apply to mathematical atomism, which asserts that the continuum is composed of an infinite number of non-extended, and hence absolutely indivisible, points.[73]

Although I am convinced that Galileo's shift from a physical to a mathematical atomism was motivated by theoretical, rather than religious, motives, I agree with Redondi that the *Labyrinthus* may have been a source of inspiration for Galileo. After all, Fromondus considered mathematical atomism to be not only less dangerous than physical atomism from a religious point of view, but also theoretically more coherent. It is worth noting that in describing the advantages of his new matter theory, Galileo refers precisely to the rarefaction and condensation of bodies, which Fromondus regarded as an insuperable challenge for physical atomism. In the first day of his *Two New Sciences,* Galileo argues that physical bodies are composed of an infinite number of non-extended atoms, which can be filled either with matter or void. While condensed bodies only contain 'filled' atoms, rarefied bodies include an infinite number of non-extended voids. As we shall see in the next section, in the last chapter of the *Labyrinthus*, Fromondus uses the example of an expanding circumference in order to explain how his own theory of the composition of continuous magnitudes permits him

[70] For the possible influence of Fromondus on Galileo, see Pietro Redondi, 'Atomi, indivisibili e dogma', *Quaderni Storici*, vol. 20, 1985, pp. 529-571 (555-557).

[71] Cf. Pietro Redondi, *Galileo Heretic*, transl. Raymond Rosenthal, Princeton, 1987.

[72] *Ibid.,* p. 26.

[73] 'Censuerunt enim Concilij Patres, compositionem ex partibus infinitis nullam involvere contradictionem, ideoque sine omnipotentiae divinae detractione & iniuria negari non posse, Deo esse possibilem, negabant tamen esse possibilem Wicleffus, Hussus & Pragensis, affirmantes nullam posse esse lineam, nisi ex punctis finitis, nec ullum tempus, nisi ex instantibus finitis immediatis', Fromondus, *Labyrinthus*, p. 12.

to account for rarefaction and condensation without admitting either void spaces or the interpenetration of matter. A similar point is made by Galileo, who after having explained the rarefaction and condensation by analogy with the behaviour of two concentric circumferences rolling on their respective tangents (the so-called *Rota Aristotelis* paradox), observes:

> 'The compacting of infinitely many unquantifiable parts without interpenetration of quantified parts, and the […] expansion of infinitely many indivisibles with the interposition of indivisible voids, I believe to be the most that can be said to explain the condensation and rarefaction of bodies without the necessity of introducing interpenetration of bodies and quantified void spaces.'[74]

Although Galileo manages to circumvent what Fromondus regarded as the main problem of physical atomism, he cannot provide an answer to the *Labyrinthus*' chief objection against mathematical atomism: that where there is no extension, there can be no matter.[75]

As shown in this section, in the fourth and fifth decades of the seventeenth century a number of works were published which championed the three variants of atomism criticized in the *Labyrinthus*. It is now time to deal with Fromondus' own theory of the composition of continuous magnitudes and to find out which explanation he proposed for the two problematic phenomena of the rarefaction and condensation of matter and of the acceleration and deceleration of motion.

6. Fromondus' theory of the composition of continuous magnitudes

Although many of the *Labyrinthus*' anti-atomistic arguments deal with circular or spherical forms, Fromondus wonders at a certain point 'whether any globe, or circle, *if it can be given*, touches a line in a single point'. Fromondus' hesitation should not surprise us, as the truth of the proposition 'a sphere touches a plain in a point' had been put in doubt by Aristotle himself. In the third book of the *Metaphysics* we read that

> 'sensible lines are not like those of which the geometrician speaks (since there is nothing sensible which is straight or curve in that sense; the circle touches the ruler not at a point, but along a line as Protagoras used to say in refuting the geometricians).'[76]

[74] Galilei, *Two New Sciences* (as in n. 69), p. 57.

[75] Fromondus, *Labyrinthus*, pp. 97-99 (Caput XXXI, *Frustra quidam conati inter Aristotelem & Epicurum medij incedere, negando ullas esse in continuo partes, aut asserendo infinitas, sed indivisibiles*).

[76] *Metaphysics,* III.2, 998a1-5. See also *De anima,* I.1, 403a10-15.

In the medieval period, the sphere-and-plane example was used by Henry of Harclay and Walter Chatton to prove the composition of continua out of indivisibles. A sphere and a plane – so Harclay and Chatton believed – touch each other at one point. Therefore, if the sphere rolls across the plane, it will touch it continuously point after point; the line described by the sphere at the end of its revolution will thus be composed of points.[77]

The touch-at-a-point argument challenged the views of medieval nominalists such as William of Ockham, Adam Wodeham and John Buridan, who denied both the composition of continuous magnitudes out of indivisibles and the existence of indivisibles in the physical world. As Jack Zupko has observed, this 'non-entitist form of divisibilism […] was less common than the orthodox Aristotelian variety […] according to which indivisibles are to be understood as real limits, though not as constituent parts of continua'.[78] The need to solve the problem without positing non-extended entities led the nominalists to develop highly sophisticated theories about the structure of the continuum.[79]

That Fromondus, like the medieval nominalists, propagates a non-entitist form of divisibilism becomes clear in chapters 32-38 of the *Labyrinthus,* which are meant to show that indivisibles can neither compose the continuum, nor function as positive termini. In these chapters Fromondus criticizes a number of views held by contemporary Aristotelians, thereby revealing that the anti-atomists are as divided as the atomists.

The first Aristotelian author with whom Fromondus takes issue is Francisco Suarez, who in the fortieth of his *Metaphysical Disputations* asserts the existence of *indivisibilia terminantia* (i.e., points ending a line, lines ending a surface, and surfaces ending a solid) and of *indivisibilia continuantia* (i.e., indivisibles joining the separable parts of a continuum).[80] Following Durandus de Sancto Porciano and Gregory of Rimini, Fromondus claims that points can only be defined privatively: they neither compose nor terminate a line, nor do they form a link between the different parts of it.[81] If God were to destroy all points joining the

[77] J. Zupko, 'Nominalism Meets Indivisibilism', *Medieval Philosophy and Theology,* vol. 3 (1993), pp. 158-185 (159-160).

[78] *Ibid.,* p. 164, n. 18.

[79] See *ibid.,* pp. 164-185, where the solutions of Ockham, Wodeham, and Buridan to the touch-at-a-point argument are analysed.

[80] For Suarez' theory of the composition of the continuum, see J. Secada, 'Suarez on Continuous Quantity' in: B. Hill and H. Lagerlund, eds, *The Philosophy of Francisco Suarez,* Oxford, 2012, pp. 75-86.

[81] 'Indivisibilia terminantia reiicit Durandus, quia inutilia sunt […]. *Fatuum enim est,* inquit, *imaginari, quod nisi essent puncta terminantia lineam, linea ex utraque parte efflueret in infinitum* […]. Nominales superficiem cum omnibus universim terminis positivis a corpore resecant. Audite

parts of a line, or the external surface of a sphere, neither the line nor the sphere would become smaller.[82]

To Suarez' claim that 'a body missing an intrinsic terminus is not apt to have either physical contact with other bodies, or a figure or other similar accidents',[83] Fromondus replies that quantities, like material qualities, can be positively unterminated (*positive interminata*). In a two-coloured piece of cloth, for example, it would be impossible for one and the same line to be the terminus of a red and a green stripe, which means that the two colours touch each other, and yet do not have a positive terminus.[84]

But if one denies the existence of indivisibles, how can one explain the contact between a globe and a plane? According to Fromondus, Aristotle believed that 'while in geometry, where abstraction is made from the sensible matter, a plane is touched by the sphere at a point [...], if one considers how the contact is exerted in a natural body, it happens instead in a divisible part'.[85] During the Renaissance debate *de certitudine mathematicarum*, this position was endorsed by Alessandro Piccolomini and Benedictus Pereira, who both invoked the proposition that 'the sphere touches the plane in a point' as a typical example of a mathematical proposition which does not apply to the physical world.[86]

inter eos facile subtilissimum. *Dico, quod nulla entitas extra animam est in magnitudine simpliciter indivisibilis, qualem existimant punctum; nec aliqua divisibilis tantum secundum unam dimensionem, qualem dicunt esse lineam, nec aliqua divisibilis tantum secundum duas, quale opinatur esse superficiem*', Fromondus, *Labyrinthus,* pp. 102-104. The first passage quoted by Fromondus is taken from *Durandi a Sancto in Petri Lombardi sententias theologicas commentariorum libri IIII*, Venice: Guerra, 1571 (reprint Ridgewood, N.J., 1964), *Liber II*, *distinctio 2*, *quaestio 4*, p. 133 v.; the second passage is taken from *Gregorii Arminensis OESA Lectura super primum et secundum Sententiarum,* Paris, 1482, *liber II, distinctio 2, quaestio 2,* reprinted in *Gregorii Ariminensis* OESA *Lectura super Primum et Secundum Sententiarum*, A. Trapp ed., Berlin and New York, *Tomus IV: Super Secundum (Dist. 1-5)*, 1979, pp. 339-343.

82 Fromondus, *Labyrinthus,* p. 105.

83 '[...] corpus carens intrinsico termino non esset aptum ad Physicum contactum cum aliis corporibus, neque ad figuram, et alia similia accidentia', F. Suarez, *Metaphysicarum disputationum, in quibus et universa naturalis theologia ordinate traditur, et quaestiones omnes ad duodecim Aristotelis libros pertinentes accurate disputantur,* first ed. Salamanca, 1597, Disputatio 40, sectio 5, quoted in *Labyrinthus,* p. 102.

84 Fromondus, *Labyrinthus,* p. 103.

85 'Geometrae punctum aliquod in circumferentia sphaerae suae, a materia sensibili abstractae in quo tangatur ab ea planum; cum tamen si consideretur, ut revera in corpore naturali tactus ille exercetur, potius fiat in parte divisibili,', *ibid.,* p. 100. For the distinction between a material and a geometrical sphere, see Aristotle, *On the Soul,* 1.1 (403 a, 12-16); *Metaphysics,* 3.2 (997b 35-998a 5); *De caelo,* 3.8.

86 For an in-depth analysis of the views of Piccolomini and Pereira on the certitude of mathematics, see Anna De Pace, *Le matematiche e il mondo: Ricerche su un dibattito in Italia nella seconda meta del Cinquecento,* Milan, 1993.

It should be clear that Fromondus, who devotes many chapters of the *Labyrinthus* to demonstrating the incompatibility between Epicurean atomism and Euclidean geometry, cannot agree with those Aristotelians who deny the applicability of mathematics to the physical world. In open opposition to Paolo Aresi, whose opinion agrees with that of Alessandro Piccolomini, Fromondus argues that God must be able to create a perfect sphere (like that of the fixed stars) and a perfect plane and then make the two figures touch one another.[87]

In the context of his discussion of the sphere and plane argument, Fromondus tries to defend the nominalist theory of the composition of the continuum against the objections formulated by Suarez and his fellow Spanish Jesuits Gabriel Vasquez, Francisco Murcia and the *Conimbricenses*. He accurately points out that while Suarez, Vasquez and Murcia ascribe the same ontological status to points, lines and surfaces, the *Conimbricenses*, following Pedro da Fonseca, explicitly assert the reality of the limiting surfaces of physical bodies, but are hesitant as far as points and lines are concerned. Fromondus is obviously thinking of that passage in book IV, chapter II, *quaestio* I of the *Commentary on Physics,* in which the *Conimbricenses* observe that while the surfaces of bodies are the seat of physical qualities, points and lines inhere in bodies 'not truly and really, but only mathematically speaking' ('non vere et realiter, sed consideratione mathematica'). [88]

In this *quaestio,* a central role is played by the touch-at-a-point argument. The *Conimbricenses* discuss two different solutions proposed by the nominalists. According to the first, the contact between the sphere and the plane is not real, in the sense that there is always an intermediate body between them; according to the second, the two figures touch each other 'indivisibly, but yet at a divisible part'.[89] After rejecting both nominalist solutions, the *Conimbricenses* come to

[87] Fromondus (*Labyrinthus*, p. 111) refers to *Pauli Aresii in Aristotelis libros De Generatione et corruptione notationes ac disputationes*, Milan: G. Bordone, 1617, *Liber I, disputatio 2, quaestio 23, sectio 7,* pp. 195-196.

[88] In this *quaestio,* the *Conimbricenses* discuss three theories *de compositione continui*. The first is that of Ockham and of other nominalists, who firmly deny the existence of indivisibles; the second view holds that indivisibles are actually present in physical bodies, either 're ipsa', or 'sola virtute divina'; the third opinion is that of Pedro da Fonseca, who accepts the existence of surfaces, but rejects that of points and lines. Though the second and the third positions are both regarded by the *Conimbricenses* as probable, they seem to incline towards the third. See *Commentarii Collegii Conimbricensis in octo libros Physicorum Aristotelis* (as in n. 18), p. 178.

[89] These two solutions, which the *Conimbricenses* ascribe to Gregory of Rimini, were also endorsed by Ockham. The first is found in his *Expositio Physicorum* and the second in *Quodlibeta septem* (see Zupko, 'Nominalism', pp. 164-169). It was, however, Wodeham who 'by means of the quasi-mathematical technique known as proportional division *ad infinitum*' explained 'how the sphere and the plane can be said to touch by means of a divisible, but in the manner of an indivisible' (*ibid.,* pp. 184-185).

the conclusion that the sphere and the plane do touch each other at an indivisible point. The contact, however, must be said to be negative, for a point is nothing other than the negation of the further extension of lines ('negatio ulterioris protensionis linearum').[90]

In the lengthy chapter 34 of the *Labyrinthus,* Fromondus tries to prove that a sphere and a plane cannot touch each other either at a determinate part or at a point. The contact cannot take place at a determinate part, since it can always be divided into two more parts, one of which is closer to the plane than the other.[91] It is, however, also impossible for the sphere and the plane to touch at a point; for if God were to destroy the point, leaving the sphere and the plane intact, the two figures would still be as close as they were before.[92] Fromondus therefore proposes to solve the sphere-and-plane problem in the fashion of those *Nominales* who said that the two figures 'touch each other in an indeterminate part', which means that they are the closest they can be without compressing or penetrating each other.[93] As Zupko has shown, there was no consensus among medieval nominalists as to how to define contact among indivisibles. While Chatton defined 'contact' as the absence of an intermediate body, Wodeham claimed that 'the sphere and plane are said to touch if they extended towards each other as far as possible without compression or penetration, regardless of presence of other, intermediate bodies (e.g., the air) between them'.[94] Fromondus, as we have seen, clearly sides with 'those nominalists' (*illi nominales*) who proposed a positive definition of 'contact'.

He regards his own solution to the sphere-and-plane problem as valid both in the realm of physics and in that of mathematics. Points are nothing but negative entities, which cannot be said to exist either in the abstract or in the concrete, either in potency or in act. All we can say about a continuous magnitude (whether

[90] *Commentarii Collegii Conimbricensis in octo libros Physicorum Aristotelis,* pp. 178-179.

[91] Fromondus, *Labyrinthus,* p. 112.

[92] A similar argument was proposed in the fourteenth century by Adam Wodeham in the *Tractatus de indivisibilibus*. See C. Grellard, 'Thought Experiments in Debates on Atomism', in: K. Ierodiakonou & S. Roux, eds, *Thought Experiments in Methodological and Historical Contexts,* Leiden, Boston, 2011, pp. 65-79 (72-74).

[93] '... hoc credo voluisse illos Nominales, qui dixerunt planum a globo in parte indeterminate tangi [...]. Facile reponi ei [=Gabriel Vazquez, who criticises the nominalists solution] a Nominali potest: tactum esse realem, quia est realis relation propinquitatis tantae, quanta inter ista corpora salvis figures, & sine penetration, esse potest', *ibid.* Fromondus criticises the view expressed by Vasquez in *Commentariorum ac disputationum in primam partem summae theologiae sancti Thomae Aquinatis, tomus secundus,* Venice: Deuchinus, 1608, *disputatio 192, caput 3,* p. 413.

[94] Zupko, 'Nominalism', p. 183.

space, time or motion) is that it is divisible into parts which are themselves always divisible.

Fromondus is, however, fully aware that Spanish Jesuits have formulated a number of important arguments in favour of the existence of indivisible terms/units. They maintain that without points, lines and surfaces, it seems impossible to account for phenomena such as the uniform increase and decrease of qualities, the formation of colour on the surface of opaque bodies, the location of the soul in the body, the motion of angels and the occurrence of instantaneous actions. Fromondus discusses these arguments one by one, coming each time to the conclusion that the phenomenon under discussion can be accounted for by postulating the composition of the continuum out of ever divisible parts. Of these arguments, Fromondus regards one as particularly challenging. Although the reflection of light rays by the external surface of opaque bodies seems to indicate that colours 'reside' in an indivisible magnitude, it is still necessary, according to him, to ascribe a certain thickness to that surface. Opacity cannot, in fact, be the quality of an indivisible surface, which must of necessity be transparent, but instead originates from the unequal arrangement of the internal parts of the body against which the light rays strike. He therefore concludes that light rays do penetrate beyond the surface of a body and that the colours which we perceive cannot be sustained by a mathematically indivisible surface.[95] As for the other arguments, Fromondus claims that nothing real can be located in an indivisible point of space or happen in an indivisible instant of time. Time, like space, is infinitely divisible, and there is no need to posit indivisible terms.

Although in the concluding chapters of the *Labyrinthus*, Fromondus champions the view that continuous magnitudes are divisible into ever divisible parts, he is not willing to endorse the Aristotelian claim that these parts are infinite in potency, but finite in act. The dichotomy finite-in-act/infinite-in-potency entails the priority of the whole above the parts, as a finite whole is potentially divisible into, but cannot be actually composed of, infinite parts. Fromondus seems to be convinced that the only way to refute physical and mathematical atomism is to propose an alternative theory of the *composition*, rather than *division*, of continuous magnitudes. This is why he argues that space, time, and matter are composed of an actual infinity of *partes proportionales*, proceeding towards ever smaller magnitudes as 1, ½, ¼, and so on. Importantly, he specifies that these parts are *non-communicantes,* that is, extrinsic to one another, like the segments AD, DF and FH in Figure 5, rather

[95] Fromondus, *Labyrinthus,* pp. 148-150.

than *communicantes*, that is, contained within one another, such as AC, AH, AF and AD.[96]

Figure 5: a finite line is composed of an infinite number of proportional parts

Being extrinsic to one another, the proportional parts of space must necessarily be traversed one after the other, that is, in the successive proportional parts of time.[97]

But how is it possible for a body to traverse an infinite number of extended parts of space in a finite time? This question is addressed by Fromondus in chapter 43 of the *Labyrinthus,* which is devoted to Zeno's paradox:

> 'If a given space were traversed with such a motion that it would take the same time to traverse individual proportional parts of space (so that one hour would be used up in the first foot, and in the following half foot, and then in the fourth part of a foot, and so on …), there is no doubt that a distance of two feet could not be exhausted. The reason is that just as the proportional parts diminish slowly *ad infinitum,* so concomitantly the speed of this motion is also remitted and diminishes.'[98]

Having shown that if the speed were 'relaxed and rarefied in this way', no distance could be exhausted in a finite time, Fromondus goes on to argue that:

> 'if the speed of motion did not decrease proportionally with the parts of space, but either remained the same, or decreased according to another law […] the entire space of two feet would be traversed in two hours, or three, or in a longer, but still finite, time.'[99]

[96] *Ibid.,* pp. 150-153. For the distinction between *partes communicantes* and partes *non communicantes*, see J.M.M.H. Thijssen, 'David Hume and John Keill and the Structure of the Continua', *Journal of the History of Ideas,* vol. 53 (1992), pp. 271-286 (280, n. 34).

[97] 'Si spatij quod pertransitur magnitudo habet partes proportionales infinitas, quarum una prior est, altera posterior, igitur corpus quod sine replicatione per tale spatium movetur, debet infinitas partes transire, unam post alteram, partesque in eo motu successivae erunt etiam infinitae: nam unicuique parti spatij permanentis, sua pars motus successivi respondet', Fromondus, *Labyrinthus,* p. 137.

[98] 'Inprimis, si spatium tali motu deberet pertransiri, qui in singulis partibus spatij proportionalibus parem moram traheret (ut si in uno pede hora consumat, & in sequenti pedis dimidio tantumdem, ac deinde in quarta pedis parte, & sic sine eundo per singulas partes proportionales una hora) proculdubio numquam eiusmodi motu spatium bipedale exhauriri posset. Ratio est, quod sicut partes proportionales gradatim in infinitum decrescunt; ita pari passu remittitur & decrescit celeritas talis motus, cum in qualibet parte proportionali sequente, duplo tardior, quam in praecedente, sit', *ibid.,* p. 154.

[99] 'Si vero celeritas motus non ita proportionaliter cum spatij partibus decrescat, sed vel eadem permaneat, vel alia lege diminuatur [...] duabus horis, aut tribus, aut pluribus (si maius decrementum tum celeritatis fuerit) finitis tamen, spatium totum bipedale emensum erit', *ibid.,* pp. 154-155.

Probably fearing that his solution to a centuries-old puzzle might appear too simple, he raises a possible objection: How can a body enter in motion, if the distance to be traversed has neither a first indivisible term or a first part? He replies that:

> 'it is possible to find an entrance, for that foot of space does have a beginning in which it is possible to enter, although none of its parts is by itself the first one. For a motion to be able to start from a first terminus, it is, in fact, sufficient that a distance is terminated by an extrinsic boundary, and that it is possible to assign a given extension, beyond which it does not stretch.'[100]

Finally, Fromondus explains that Zeno's paradox arises from a mistaken understanding of the nature of space, time and motion. The fact that space is a permanent quantity, whereas time is a successive one, does not mean that there cannot be a one to one correspondence between the infinite proportional parts of a finite line and the infinite proportional parts of a time interval.

The motion of a body along the infinite proportional parts of a line must not be compared 'with that of infinite feet of water which can never flow in a river bed, but rather with one sole foot of water, which contains infinite proportional parts of water, all of which can however flow by the shore in a very brief time.'[101] In these lines, Fromondus seems to hold the view, which will be later defended by Newton and Leibniz, that all continuous quantities are generated by motion and hence are necessarily dependent on time.

As we have seen in the previous section, Fromondus shared the belief of many early modern natural philosophers in the isomorphism of space, time and matter. It is therefore not surprising to see that he establishes a direct connection between the 'rarefaction of motion', as he calls it, and the rarefaction of matter. In the last chapter of the *Labyrinthus,* Fromondus argues that, although in the natural course of things bodies do not rarefy *ad infinitum,* the composition of the continuum out of an infinite number of proportional parts allows for the possibility that God, in his omnipotence, may produce such an endless rarefaction.

> 'God can endlessly rarefy matter, either without form or with a form preternaturally conserved. This can be shown by means of the example of the slowness of motion:

[100] 'Respondeo tamen, facile fuisse introitum reperire, quia spatium illud pedale principium aliquot habet, quo iniri possit, etiamsi nulla eius pars per se primo prima sit. Sufficit tamen spatium illud fine extrinseco terminatum esse, & extensionem aliquam posse assignari, ultra quam non excurrit, ut motus valeat ab eo termino inchoari', *ibid.,* p. 155.

[101] 'Sed fallitur. Non enim ille unius horae motus debet comparari cum infinitis aquae pedibus, qui numquam alveo fluminis effluere possent, sed cum unico potius aquae pede, qui infinitas partes aquae proportionales continet, & tamen omnes successive brevissimo tempore ripam praeterlabuntur', *ibid.,* p. 156.

> for slowness, being a laxness of parts in successive continua, is very similar to rarity in permanent continua. […] And just as from the infinite slowness of motion it can be shown that rarefaction can proceed endlessly, so from the speed of motion, which can also be increased to infinity (for could not God create celestial spheres that were bigger and bigger *ad infinitum* and yet able to accomplish a rotation in 24 hours?), we can show that also condensation, if related to divine power, can be without end.'[102]

In this case, as in many others, Fromondus feels the need to explain how his own theory differs from those of other scholastic authors. He refers to Marsilius of Inghen and Francisco Suarez who, in his view, fail to explain how condensation can take place without an interpenetration of parts or loss of quantity, and rarefaction without an intermixture of void particles or an increase of quantity.[103] Fromondus' own explanation of the rarefaction and condensation rests on the scholastic distinction between 'real' extension (*extensio realis*), that is, the tri-dimensional space actually occupied by a body, and 'aptitudinal' extension (*extensio aptitudinalis*), which is a body's aptitude for occupying a tri-dimensional space. Fromondus claims that all the infinite proportional parts composing a body can acquire a bigger or smaller real extension while retaining the same aptitudinal extension. To those who object that an increase of real extension must necessarily cause interpenetration, he replies that this would only be the case if the real extension of the entire body remained the same, which is obviously not the case in rarefaction. Once again Fromondus strengthens his point by means of a comparison with local motion:

> 'The parts which in a slow motion succeed each other more loosely (and yet without intermixed pauses), in a fast motion follow each other more closely and close their ranks; and yet they do not penetrate each other in the same time, for otherwise they would not be successive, but coexistent parts […]. Hence, if it is possible to understand how the flux of a slow motion can be accelerated, it should not be impossible

[102] 'Deus tamen materiam illam sine forma, aut cum ipsa contra naturam conservata, potest sine fine rarefacere. Quod etiam exemplo tarditatis in motu possumus declarare: tarditas enim in successivis est quaedam partium laxitas, simillima raritati in permanentibus. […]. Veluti autem ex *tarditate* motus infinita *rarefactionem* posse sine fine procedere ostenditur; ita ex *velocitate* motus, quae etiam in infinitum increscere potest (cur enim Deus caelestes sphaeras ampliores & ampliores sine fine creare nequeat, quae omnes 24 horarum spatio revolvantur?) ostendere possumus, *condensationem*, si ad virtutem divinam comparemus, nullum habere finem', *ibid.,* pp. 191-193.

[103] *Ibid.,* pp. 167-168. For Marsilius of Inghen's account of rarefaction and condensation, see E. Grant, ed., *A Sourcebook in Medieval Science,* Cambridge, MA, 1974, pp. 350-352. For Suarez'explanation of rarefaction and condensation, see R. Pasnau, *Metaphysical Themes 1274-1671*, Oxford, 2011, pp. 312-314.

to understand how a more rarefied continuous quantity can be compressed and condensed without penetration.'[104]

Like his atomist adversaries, Fromondus thus regards deceleration as the key to understanding rarefaction and acceleration as the key to understanding condensation.

7. Conclusion

At first glance, the *Labyrinthus* might seem to be a rather unoriginal piece of work. As we have seen, Fromondus devotes two-thirds of the book to summarizing traditional objections to atomism, which he supplements with some arguments of his own. He eagerly draws on a variety of sources in his attempt to show that atomism was considered untenable by the majority of ancient and medieval natural philosophers. The most interesting aspects of the *pars destruens* of the *Labyrinthus* are precisely this somewhat biased reconstruction of the history of atomism, as well as the connection which Fromondus establishes between the two apparently unrelated phenomena of the rarefaction and condensation of matter, on the one hand, and the acceleration and deceleration of motion, on the other. The short *pars construens* of the book is far more original, for Fromondus attempts to distinguish his position from that of his Aristotelian, especially Jesuit, contemporaries. He challenges the views of Piccolomini and Aresius that mathematical truths do not apply in the physical realm; he criticizes Suarez, Vasquez, Murcia, Fonseca and the *Conimbricenses* for postulating the existence of positive indivisibles in the continuum; and he even distances himself from Aristotle's view that the infinity of parts in the continuum is merely potential. Fromondus argues, following the medieval nominalists, that indivisibles are nothing but negative entities, which are found neither in the mathematical nor in the physical realm. According to him, continuous magnitudes are not only potentially divisible into, but actually composed of, an infinity of proportional parts.

A similar position was defended by Isaac Barrow in first of his *Mathematical Lectures*, which Newton attended in 1665. After having explained the difference between what we would nowadays call divergent and convergent infinite series, Barrow claimed that not only in arithmetic, but also in geometry, it was not

[104] 'In velocitate, inquam, partes quae in motu tardiori laxius (nullis tamen pausis interiectis) sibi succedunt, pressius ita sequuntur & agmen densant, ut tamen se in eodem tempore non penetrent, alias enim non partes successivae, sed coexistentes essent […]. Si igitur intelligi potest, quomodo continuus motus tardioris fluxus possit accelerari, nec impossibilis erit aliquis intellectus, quo quantitas continua rarior comprimi valeat & densari sine penetratione', Fromondus, *Labyrinthus,* pp. 175-176.

impossible for a finite magnitude to be composed of an infinite number of proportional parts, 'as nothing holds for number, which does not at the same time also hold for magnitude, which is represented and denominated by number'.[105] Although there is no reason to think that Barrow was acquainted with the *Labyrinthus,* it is interesting to note that, like Fromondus, he used the notion of a convergent infinite series to criticize Epicurus' view that an infinity of extended parts must necessarily add up to an infinite magnitude.

As mentioned in the introduction, the only early modern author who explicitly acknowledged his debt to the *Labyrinthus* was Leibniz. This debt might, in fact, be greater than scholars have so far assumed. There exists a manuscript, entitled *Geschichte des Kontinuumproblems*, which Leibniz probably drafted in late 1693. Manuel Luna Alcoba, who transcribed this manuscript in 1996, expressed his astonishment at the many 'unexpected sources from which Leibniz' theory of the continuum are fed'.[106] Luna Alcoba failed to realize, however, that this manuscript is a set of reading notes on the *Labyrinthus*. Oddly enough, Fromondus is quoted a number of times in Leibniz' text, but there is no explicit acknowledgement that he is its main source. For a reader familiar with the *Labyrinthus*, it is evident that the opening lines are merely a dense and hasty summary of the first chapters of the treatise. There, Leibniz quotes the condemnation of Wycliff at the Council of Constance, ascribes to Plato, Pythagoras, Chrysippus, and Zeno of Elea the view that continuous magnitudes are divisible *ad infinitum,* and mentions Johannes Maior as the only medieval author to compose the continuum out of infinite points.[107] In the subsequent pages, Leibniz discusses a number of medieval and early modern solutions to the problem of the continuum.[108] Here again, he clearly draws on the *Labyrinthus,* from which he borrows references to the works of Thomas Aquinas, Duns Scotus, Gregory of Rimini, Paolo Aresi, Pedro da Fonseca and Alessandro Piccolomini. Leibniz supplements Fromondus' account with contemporary mathematical sources – he refers to Daniel Bernouilli's spiral, Evangelista Torricelli's infinitely long solid and François Viète's polemics with Thomas Hobbes – but does not mention any recent atomistic

[105] 'Quod infinita series fractionum certa qualibet proportione decrescentium æquetur certo numero, vel unitati, vel unitatis parti […] satis clare docetur et ostenditur ab Arithmetica, unde non repugnat finitum aliquod infinitas in se partes continere : praesertim cum numero nihil conveniat, quod non potiori jure convenit magnitudini, quam numerus repraesentat ac denominat', I. Barrow, *Lectiones Mathematicae xxiii; In quibus Principia Matheseôs generalia exponuntur: Habitae Cantabrigiae A.D. 1664, 1665, 1666*, London, G. Wells, 1685, p. 20.

[106] M. Luna Alcoba, 'G.W. Leibniz: Geschichte des Kontinuumproblems'*, Studia Leibniziana,* vol. 28, 1996, pp. 183-198 (183).

[107] *Ibid.,* pp. 184-185.

[108] *Ibid.,* pp. 185-188.

work. For example, when criticizing discontinuist explanations of the slowness of motion, he refers to the passage of Vallesius' *Controversiarum medicarum [...] libri decem* quoted in the *Labyrinthus,* but does not mention that a similar view was espoused in more influential works such as Gassendi's *Animadversiones* (1649) and *Syntagma Philosophicum* (1658) or Walter Charlton's *Physiologia Epicuro-Gassendo-Charletoniana* (1654).

The better part of Leibniz' manuscript is devoted to trying to answer various questions raised by Fromondus, such as the uniform increase and decrease of qualities, the composition of a line described by a circumference revolving on its tangent, and the contact between a sphere and a plane.[109] Leibniz treats the *Labyrinthus*, from which he also borrows various diagrams, as a source of problems, to which he attempts to find his own solutions; the formula 'quaeritur', 'respondeo' is repeatedly used.

It is beyond the scope of the present article to offer an exegesis of Leibniz' *Geschichte des Kontinuumproblems* and of its relation to the *Labyrinthus*. Here it must suffice to point out that an historically minded polymath such as Leibniz considered Fromondus' work to be an ideal basis for his own speculations about the continuum. Whether other mathematicians and natural philosophers used this text in a comparable way remains to be seen.

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[109] *Ibid.,* pp. 187-198.